THE EFFECTS OF THE NODAL REGRESSION OF THE ORBIT ON THE GRAVITY PRECESSION OF A GYROSCOPIC SATELLITE

James L. Myers, Jr.

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NOTATION

A	Moment of inertia about transverse axes
a	Semi-major axis of ellipse
$\hat{a}_1$, $\hat{a}_2$	Unit vectors
C	Polar moment of inertia
e	Eccentricity of orbit
G	Universal gravitational constant
$\underline{H}$	Angular momentum vector
i	Orbital inclination; angle measured from earth's polar axis to orbital angular velocity vector
$\hat{i}$, $\hat{j}$, $\hat{k}$	Unit vectors in inertial coordinate system
M	Mass of the earth
M_x, M_y, M_z	Components of gravity gradient moment
R	Radius from center of earth to center of satellite
R_e	Radius of earth
T	Orbital period
t	Time
x, y, z	Coordinate axes
α	Argument of perigee in orbital plane
δ_e	Spin axis error angle for equatorial orbit (small)
δ_p	Spin axis error angle for polar orbit
ϵ	Gyro spin axis inclination
η	Coordinate transformation angle
θ	Angle between gyro spin axis and orbital angular velocity vector

Λ	Gravity gradient precession coefficient
φ	Right ascension of gyro; angle measured west-to-east in earth's equatorial plane from the vernal equinox to the line of nodes between gyro and earth equatorial planes
Ψ	Argument of satellite in orbital plane
Ω	Right ascension of orbit
$\underline{\omega}$	Angular velocity vector of gyro precession
$\underline{\omega}_o$	Orbital angular velocity vector
$\underline{\omega}_s$	Gyro spin axis vector
$\hat{}$	Denotes unit vector

A gyroscopic satellite has been proposed for a test of the general theory of relativity[1,2] in which the gyro "drift" rate to be measured is less than seven seconds of arc per year. The Coordinated Science Laboratory has proposed a simple passive gyroscope which uses sunlight reflected from mirrors to provide optical data to determine the spin axis orientation.[3,4] The gravity gradient torque acting on the satellite is one of several extraneous disturbances which can cause spurious precession of the gyro spin axis. In this paper, general equations for the precession of a gyro satellite in a regressing orbit are derived. These equations may be used to specify the tolerances for initial spin axis and orbit alignments which enable an accurate measurement of the relativity effect.

1. Gravity Gradient Moment

The gravity gradient moment is given by ref. 5 for the fixed orbit configuration shown in Fig. 1. Two coordinate systems are shown, system [2] fixed to the body spin axis, $\underline{\omega}_s$, and system [3] fixed to the orbital plane, or to the orbital angular velocity vector, $\underline{\omega}_o$. The x_2 axis is the line of nodes between the orbital (x_3 - y_3) plane and the equatorial (x_2 - y_2) plane of the orbiting gyro. The gravity gradient moment components in the body axis system given in ref. 5 are

$$\left.\begin{aligned} M_{x_2} &= -\frac{3GM}{R^3}(C-A)\sin\theta\cos\theta\sin^2\omega_o t \\ M_{y_2} &= -\frac{3}{2}\frac{GM}{R^3}(C-A)\sin\theta\sin 2\omega_o t \\ M_{z_2} &= 0 \end{aligned}\right\} \qquad (1)$$

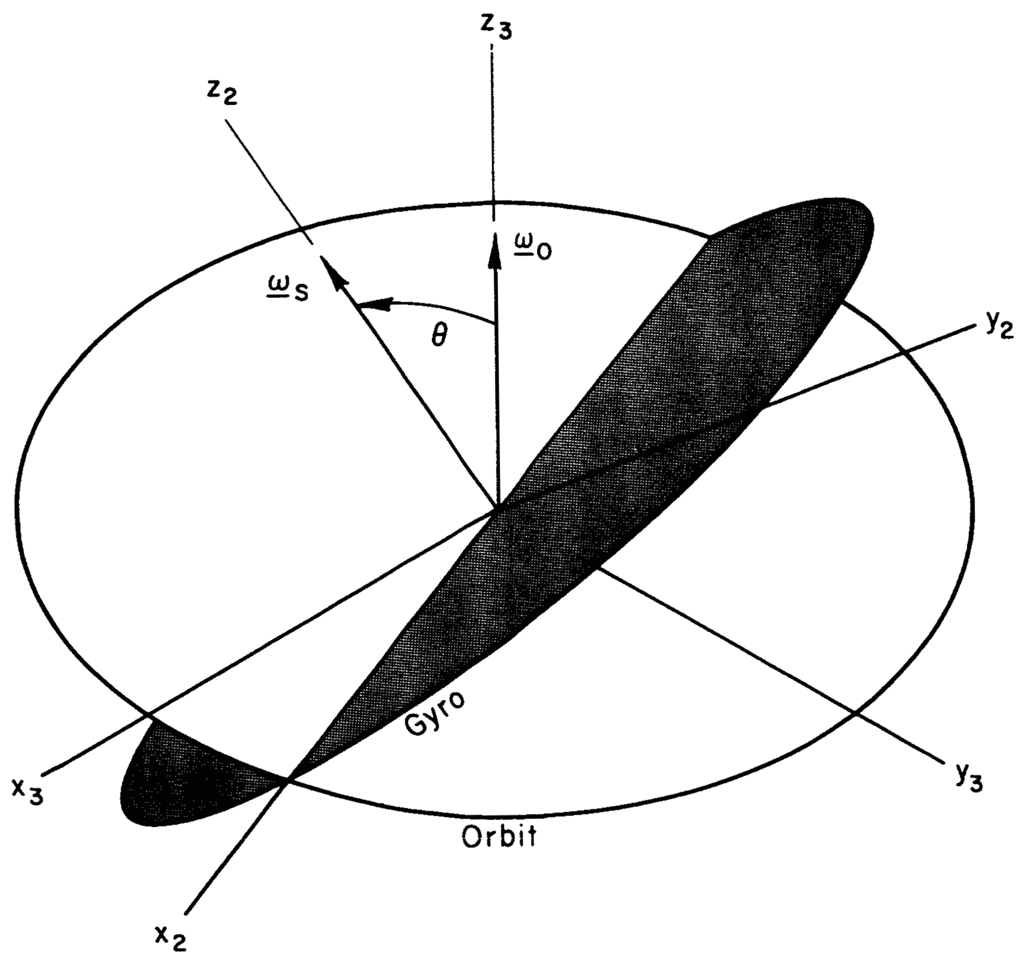


Fig. 1 Planes of gyro equator and orbit; coordinate systems [2] and [3].

where M is the mass of the earth and R is the distance between the centers of the satellite and the earth. θ is the angle between the vectors $\underline{\omega}_s$ and $\underline{\omega}_o$, and varies only because of precession of $\underline{\omega}_s$. In the general case the satellite's orbital plane will regress about the earth's polar axis at the rate of $\dot{\Omega}$ degrees per year[6] and will produce an additional change in θ.

The geometry involved in the general case of interest for regressing orbits is illustrated in Fig. 2. An earth-based coordinate system is fixed with z_o along the earth's north pole and x_o along the line of vernal equinox (i.e., the line of the nodes between the ecliptic and the earth's equatorial plane). The y_o axis completes an orthogonal right-handed system and therefore, lies in the earth's equatorial plane. Systems [2] and [3] bear the same relationship to each other as shown in Fig. 1. A new coordinate system, [1], is shown with z_1 also along the gyro spin axis, but with x_1 along the line of nodes between the earth's and gyro's equatorial planes. This is the most logical system to observe gyro motion with respect to the earth. The moment given for system [2] can be transformed through angle η to system [1] by the transformation

$$\begin{bmatrix} \cos\eta & -\sin\eta & 0 \\ \sin\eta & \cos\eta & 0 \\ 0 & 0 & 1 \end{bmatrix} .$$

The moment components in system [1] are now given by

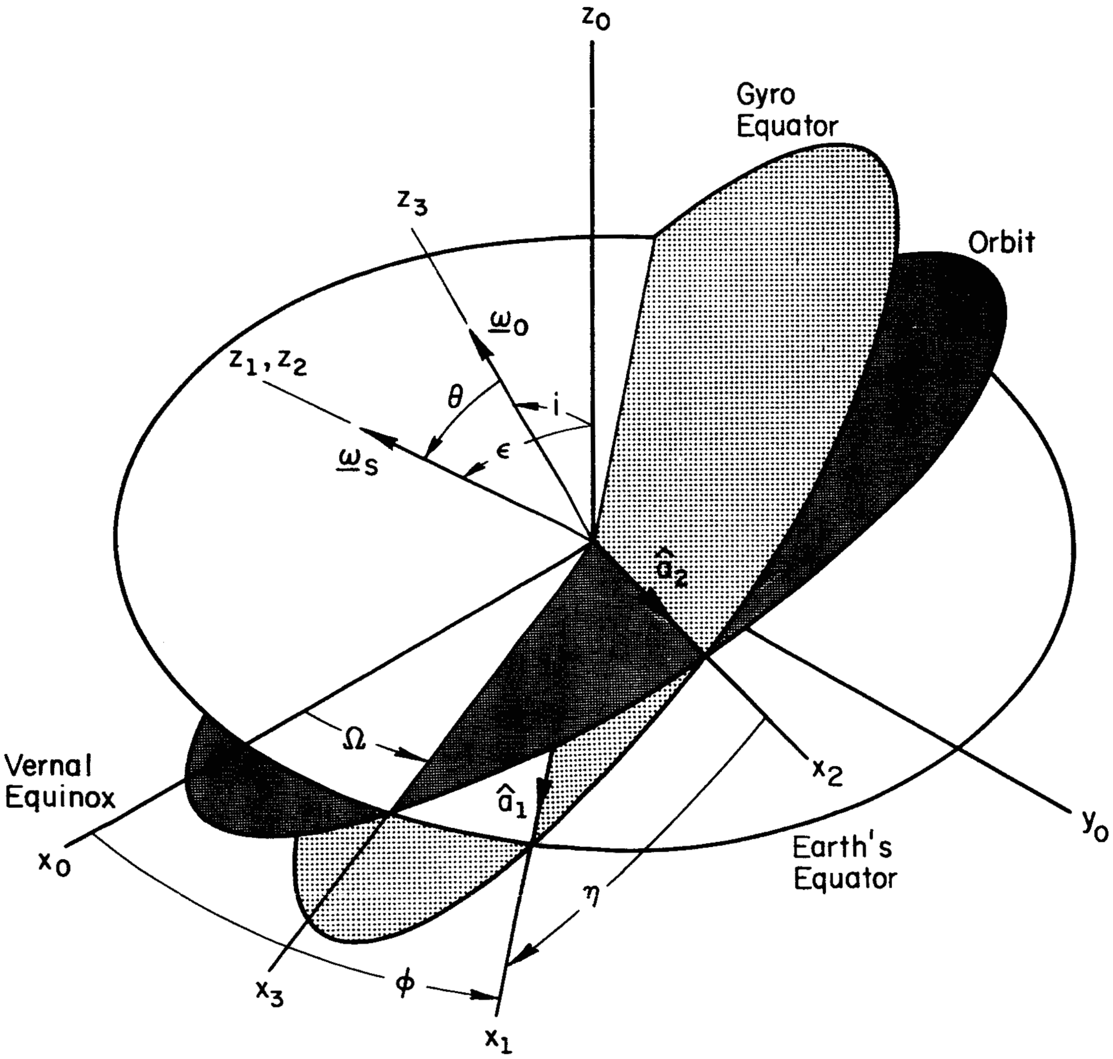


Fig. 2 Orbit and gyro frames related to inertial system; coordinate systems [0], [1], [2], [3].

$$M_{x_1} = \cos\eta\, M_{x_2} - \sin\eta\, M_{y_2}$$

$$M_{y_1} = \sin\eta\, M_{x_2} + \cos\eta\, M_{y_2}$$

$$M_{z_1} = M_{z_2} = 0 .$$

Substituting the values for M_{x_2} and M_{y_2} gives

$$M_{x_1} = -\frac{3GM}{R^3}(C-A)\cos\eta\sin\theta\cos\theta\sin^2\omega_o t + \frac{3}{2}\frac{GM}{R^3}(C-A)\sin\eta\sin\theta\sin 2\omega_o t \tag{2}$$

$$M_{y_1} = -\frac{3GM}{R^3}(C-A)\sin\eta\sin\theta\cos\theta\sin^2\omega_o t - \frac{3}{2}\frac{GM}{R^3}(C-A)\cos\eta\sin\theta\sin 2\omega_o t. \tag{3}$$

$\cos\theta$, $\cos\eta\sin\theta$, and $\sin\eta\sin\theta$ will now be found as functions of i, Ω, ϵ, and φ. From Fig. 2, the gyro spin axis unit vector is given as

$$\hat{\omega}_s = \sin\epsilon\sin\varphi\hat{i} - \sin\epsilon\cos\varphi\hat{j} + \cos\epsilon\hat{k}, \tag{4}$$

and the orbital angular velocity unit vector is

$$\hat{\omega}_o = \sin i\sin\Omega\hat{i} - \sin i\cos\Omega\hat{j} + \cos i\hat{k}. \tag{5}$$

Therefore, $\cos\theta = \hat{\omega}_s \cdot \hat{\omega}_o$

$$= \sin i\sin\epsilon\cos(\Omega-\varphi) + \cos i\cos\epsilon. \tag{6}$$

From Fig. 2 a unit vector $\hat{a}_1$ is defined along x_1:

$$\hat{a}_1 = \cos\varphi\hat{i} + \sin\varphi\hat{j}.$$

From Fig. 2 it is seen that the x_2 axis is the line of nodes between the gyro equatorial plane and the orbital plane. Therefore, the x_2 axis is normal to both $\hat{\omega}_s$ and $\hat{\omega}_o$, and a new vector along x_2 may be defined:

$$\underline{a}_2 \equiv \hat{\omega}_o \times \hat{\omega}_s = \sin\theta\ \hat{a}_2 .$$

Now

$$\hat{a}_1 \cdot \underline{a}_2 = a_2 \cos\eta = \sin\theta \cos\eta$$

or

$$\cos\eta \sin\theta = \hat{a}_1 \cdot \hat{\omega}_o \times \hat{\omega}_s$$

$$= - \sin i \cos\epsilon \cos(\Omega - \varphi) + \cos i \sin\epsilon . \qquad (7)$$

Also it can be seen that

$$\hat{a}_1 \times \underline{a}_2 = a_2 \sin\eta\ \hat{\omega}_s$$

$$= \sin\theta \sin\eta\ \hat{\omega}_s$$

or

$$\sin\eta \sin\theta\ \hat{\omega}_s = \hat{a}_1 \times (\hat{\omega}_o \times \hat{\omega}_s)$$

$$= (\hat{a}_1 \cdot \hat{\omega}_s)\ \hat{\omega}_o - (\hat{a}_1 \cdot \hat{\omega}_o)\ \hat{\omega}_s .$$

But,

$$\hat{a}_1 \cdot \hat{\omega}_s = 0,$$

therefore,

$$\sin\eta \sin\theta = - (\hat{a}_1 \cdot \hat{\omega}_o).$$

Substitution of the vector components yields

$$\sin\eta \sin\theta = - \sin i \sin(\Omega - \varphi). \qquad (8)$$

Equations (7) and (8) can now be substituted into the moment equations (2) and (3):

$$M_{x_1} = \frac{3GM}{R^3}(C-A)[\sin i \cos\epsilon \cos(\Omega-\varphi) - \cos i \sin\epsilon] \cos\theta \sin^2 \omega_o t$$

$$- \frac{3}{2}\frac{GM}{R^3}(C-A) \sin i \sin(\Omega-\varphi) \sin 2\omega_o t \tag{9}$$

$$M_{y_1} = 3\frac{GM}{R^3}(C-A) \sin i \sin(\Omega-\varphi) \cos\theta \sin^2 \omega_o t$$

$$+ \frac{3}{2}\frac{GM}{R^3}(C-A)[\sin i \cos\epsilon \cos(\Omega-\varphi) - \cos i \sin\epsilon] \sin 2\omega_o t \tag{10}$$

$$M_{z_1} = 0 . \tag{11}$$

These equations give the gravity gradient moment for a given set of orbital parameters, i and Ω, and gyro spin direction ϵ and φ.

2. Precession

The precession rate $\underline{\omega}$ of coordinate system [1] can be found from Euler's dynamical equation

$$\dot{\underline{H}} + \underline{\omega} \times \underline{H} = \underline{M}. \tag{12}$$

By inspection of Fig. 2, the components of $\underline{\omega}$ in system [1] are written

$$\omega_{x_1} = \dot{\epsilon}$$

$$\omega_{y_1} = \dot{\varphi} \sin \epsilon$$

$$\omega_{z_1} = \dot{\varphi} \cos \epsilon .$$

Since the coordinate axes of system [1] lie along the principal axes of the body, the angular momentum vector $\underline{H}$ and its derivative are given as

$$H_{x_1} = A\,\dot{\epsilon}$$

$$H_{y_1} = A\,\dot{\varphi}\,\sin\epsilon$$

$$H_{z_1} = C(\omega_s + \dot{\varphi}\,\cos\,\epsilon)$$

and

$$\dot{H}_{x_1} = A\,\ddot{\epsilon}$$

$$\dot{H}_{y_1} = A(\ddot{\varphi}\,\sin\,\epsilon + \dot{\varphi}\,\dot{\epsilon}\,\sin\,\epsilon)$$

$$\dot{H}_{z_1} = C(\dot{\omega}_s + \ddot{\varphi}\,\cos\,\epsilon - \dot{\varphi}\,\dot{\epsilon}\,\sin\,\epsilon).$$

Upon substitution of these components into Eq. (12) we have

$$\left.\begin{array}{ll} A\ddot{\epsilon} + C\omega_s\,\dot{\varphi}\,\sin\,\epsilon + (C-A)\,\dot{\varphi}^2\,\sin\,\epsilon\,\cos\,\epsilon & = M_{x_1} \\ (2A - C)\,\dot{\varphi}\,\dot{\epsilon}\,\cos\,\epsilon + A\ddot{\varphi}\,\sin\,\epsilon - C\omega_s\,\dot{\epsilon} & = M_{y_1} \\ C(\dot{\omega}_s + \ddot{\varphi}\,\cos\,\epsilon - \dot{\varphi}\,\dot{\epsilon}\,\sin\,\epsilon) & = M_{z_1} \end{array}\right\} . \tag{13}$$

The angular rate of the gyro, ω_s, typically is more than ten orders of magnitude larger than $\dot{\varphi}$ or $\dot{\epsilon}$. As will be seen later, $\ddot{\varphi}$ and $\ddot{\epsilon}$ are of the order of $\dot{\varphi}^2$ or $\dot{\epsilon}^2$. Therefore, Eqs. (13) may be simplified by neglecting all terms on the left-hand side which do not contain the factor ω_s. Now it can be seen that

$$\dot{\varphi} = \frac{M_{x_1}}{C\omega_s\,\sin\,\epsilon} \tag{14}$$

$$\dot{\epsilon} = -\,\frac{M_{y_1}}{C\omega_s} \tag{15}$$

$$\dot{\omega}_s = \dot{\varphi}\,\dot{\epsilon}\,\sin\epsilon - \ddot{\varphi}\,\cos\,\epsilon. \tag{16}$$

Substituting Eqs. (9) and (10) into (14) and (15) gives

$$\dot{\varphi} = \frac{3GM}{R^3\omega_s}\left(\frac{C-A}{C}\right)\{[\sin i \cot \epsilon \cos(\Omega-\varphi) - \cos i]\cos\theta \sin^2\omega_o t + \frac{1}{2}\frac{\sin i}{\sin \epsilon}\sin(\Omega-\varphi) \sin 2\omega_o t\} \tag{17}$$

$$\dot{\epsilon} = \frac{3GM}{R^3\omega_s}\left(\frac{C-A}{C}\right)\{\sin i \sin(\Omega-\varphi) \cos\theta \sin^2 \omega_o t - \frac{1}{2}[\sin i \cos \epsilon \cos(\Omega-\varphi) - \cos i \sin \epsilon] \sin 2\omega_o t\} \tag{18}$$

where $\cos\theta$ is given by Eq. (6).

Assuming that i, Ω, ϵ, and φ change much less rapidly than $\omega_o t$, average rates $\tilde{\dot{\varphi}}$ and $\tilde{\dot{\epsilon}}$ may be found by integrating over one orbital period

$$\tilde{\dot{\varphi}} = \frac{1}{T}\int_o^T \dot{\varphi}dt$$

$$\tilde{\dot{\epsilon}} = \frac{1}{T}\int_o^T \dot{\epsilon}dt.$$

For an elliptical orbit, the radius R from the center of the earth is

$$R = \frac{a(1-e^2)}{1 + e\cos(\Psi-\alpha)}$$

where
- a = semi major axis of the ellipse
- e = eccentricity
- $\Psi = \omega_o t$ = argument of the satellite
- α = argument of perigee.

Also, Kepler's law of areas provides the relation

$$R^2\dot{\Psi} = \sqrt{GM\, a(1-e^2)}$$

therefore,

$$\frac{dt}{R^3} = \frac{dt}{R\,R^2\,\frac{d\Psi}{dt}} \cdot \frac{d\Psi}{dt} = \frac{d\Psi}{\sqrt{GM\,a(1-e^2)}\,R} .$$

Now, integration of (17) over one orbital period becomes an integration from 0 to 2π in Ψ:

$$\tilde{\dot{\varphi}} = \frac{3\,GM}{\sqrt{GM\,a(1-e^2)}} \frac{C-A}{C\omega_s} \{[\sin i\ \cot \epsilon\ \cos(\Omega-\varphi) - \cos i]\cos\theta$$

$$\frac{1}{a(1-e^2)T} \int_0^{2\pi} \sin^2\Psi[1 + e(\cos\Psi\ \cos\alpha + \sin\Psi\ \sin\alpha)]d\Psi$$

$$+ \frac{\sin i}{\sin\epsilon} \frac{\sin(\Omega-\varphi)}{a(1-e^2)T} \int_0^{2\pi} \sin\Psi\ \cos\Psi[1 + e(\cos\Psi\ \cos\alpha + \sin\Psi\ \sin\alpha)]d\Psi\} .$$

The first integral yields π, and the second integral vanishes. Equation (18) is integrated similarly, and the resulting time averages are

$$\tilde{\dot{\varphi}} = \Lambda[\sin i\ \cot\epsilon\ \cos(\Omega-\varphi) - \cos i]\cos\theta \tag{19}$$

$$\tilde{\dot{\epsilon}} = \Lambda[\sin i\ \sin(\Omega-\varphi)]\cos\theta \tag{20}$$

where the gravity gradient precession coefficient is defined as

$$\Lambda \equiv \frac{3}{2} \frac{GM}{a^3(1-e^2)^{3/2}} \frac{C-A}{C\omega_s} .$$

The orbital period T, has been eliminated by the equation

$$T = \frac{2\pi\,a^{3/2}}{\sqrt{GM}} .$$

Equations (19) and (20) may be integrated with respect to time for any $i(t)$ and $\Omega(t)$ to give φ and ϵ as functions of time. $\dot{\tilde{\varphi}}$ and $\dot{\tilde{\epsilon}}$ are both of the order of Λ, and therefore, this quantity must be small (specifically, $\frac{\Lambda}{\omega_s} \ll 1$) for the foregoing derivation to be valid. Furthermore, the time derivatives of (19) and (20) show that $\ddot{\tilde{\varphi}}$, $\ddot{\tilde{\epsilon}}$, and therefore $\dot{\omega}_s$ are of the order $\dot{\tilde{\varphi}}^2$ or $\dot{\tilde{\epsilon}}^2$, as assumed previously.

3. Special Orbits

The relativity drift rate of the gyro spin axis will be largest when the spin axis lies in the orbital plane.[1] Therefore, two cases of special interest are an equatorial orbit and a polar orbit, because either of these orbits will allow the gyro spin axis to lie in the orbital plane for an extended period of time. For each of these special cases, Eqs. (19) and (20) may be simplified and integrated directly, as will be seen.

A. Equatorial Orbit

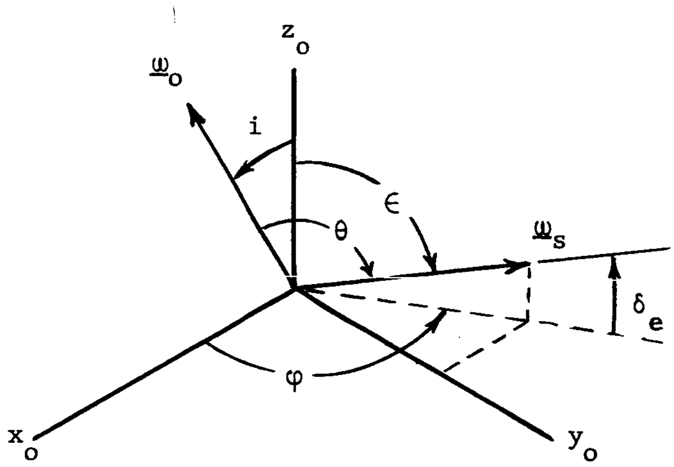
For an equatorial orbit, the inclination i will be assumed small so that

$$\sin i \approx i,$$
$$\cos i \approx 1.$$

Also, it is assumed that

$$\epsilon = \frac{\pi}{2} + \delta_e,$$

where δ_e is a small angle between the spin axis and the $x_o - y_o$ plane, as shown in the sketch.



Now,

$$\sin \epsilon = \sin(\frac{\pi}{2} + \delta_e) = \cos \delta_e \approx 1 \tag{21}$$

$$\cos \epsilon = \cos(\frac{\pi}{2} + \delta_e) = -\sin \delta_e \approx -\delta_e . \tag{22}$$

From Eq. (6)

$$\cos \theta = \sin i \cos(\Omega-\varphi) - \delta_e \cos i .$$

Since i is also a small angle,

$$\cos \theta \approx i \cos(\Omega-\varphi) - \delta_e . \tag{23}$$

Substituting Eqs. (21) to (23) into Eqs. (19) and (20) results in the following

$$\tilde{\dot{\varphi}} = - \Lambda\Big[i^2 \delta_e \cos^2(\Omega-\varphi) + i \, \delta_e^{\,2} \cos(\Omega-\varphi)$$
$$+ i \cos(\Omega-\varphi) - \delta_e \Big]$$

$$\tilde{\dot{\epsilon}} = \Lambda\Big[\frac{i^2}{2} \sin 2(\Omega-\varphi) - i \, \delta_e \sin(\Omega-\varphi)\Big] .$$

Since i and δ_e are both small, $\dot{\tilde{\varphi}}$ is larger by at least one order of magnitude than $\dot{\tilde{\epsilon}}$, and may now be simplified by dropping the higher order terms in i and δ_e:

$$\dot{\tilde{\varphi}} \approx \Lambda[\delta_e - i\cos(\Omega-\varphi)]. \tag{24}$$

For near equatorial orbits, Ω changes at the rate of 6 to 9 revolutions per year. Therefore, Eq. (24) indicates that the average rate of change of φ is proportional to δ_e, the angle between the gyro spin axis and the earth's equatorial plane. Setting $\Omega = \Omega_o + \dot{\Omega}t$, Eq. (24) can be integrated with respect to time to give $\Delta\varphi$:

$$\Delta\varphi = \Lambda\left\{\delta_e - i\left[\cos(\Omega_o - \varphi)\frac{\sin\dot{\Omega}t}{\dot{\Omega}t} + \sin(\Omega_o - \varphi)\frac{(\cos\dot{\Omega}t-1)}{\dot{\Omega}t}\right]\right\}t.$$

By arbitrarily setting $\Omega_o - \varphi = \frac{\pi}{2}$, this simplifies to

$$\Delta\varphi = \Lambda\left[\delta_e + i\left(\frac{1-\cos\dot{\Omega}t}{\dot{\Omega}t}\right)\right]t. \tag{25}$$

From Eq. (25) it is seen that for large values of $\dot{\Omega}t$, $\Delta\varphi$ is proportional to δ_e.

B. Polar Orbit

A true polar orbit (i.e., $i = \frac{\pi}{2}$) is required for a nonregressing orbit plane. The nodal regression rate of the orbit line of nodes is given in ref. 6 by

$$\dot{\Omega} = \frac{-3}{2} J_2\sqrt{\frac{GM}{a}}\left(\frac{R_e^2}{a^3}\right)\left(\frac{1}{1-e^2}\right)^2\cos i, \tag{26}$$

where $J_2 = 1.082 \times 10^{-3}$ is the coefficient of the second harmonic term in the earth's gravitational potential. (A more exact equation for nodal regression, also given in ref. 6, contains terms three orders of magnitude smaller than Eq. (26) and will not be required in this analysis.) The right ascension of the orbit line of nodes will now be written $\Omega = \Omega_o + \dot{\Omega}t$, where Ω_o is the value of Ω at the time of injection into orbit. Figure 3 shows a typical configuration for a near polar orbit right after injection. Here, δ_p is an error angle between the initial orbit line of nodes and the projection of the gyro spin axis on the earth's equatorial plane. It will be seen that the gravity gradient precession depends on this angle and on the regression rate, $\dot{\Omega}$.

From Fig. 3 it can be seen that

$$\Omega_o - \varphi = \frac{\pi}{2} - \delta_p,$$

and, therefore,

$$\Omega - \varphi = \dot{\Omega}t + \frac{\pi}{2} - \delta_p.$$

Now,

$$\cos(\Omega-\varphi) = \sin(\delta_p - \dot{\Omega}t)$$

$$\sin(\Omega-\varphi) = \cos(\delta_p - \dot{\Omega}t).$$

Also, assuming that $i = \frac{\pi}{2} + i'$, where i' is a small error in orbital inclination, we have

$$\cos\theta = \sin\epsilon\,\sin(\delta_p - \dot{\Omega}t) - i'\cos\epsilon.$$

Equations (19) and (20) now become

$$\tilde{\dot{\varphi}} = \Lambda\Big[\cos\epsilon\,\sin^2(\delta_p - \dot{\Omega}t) - i'\,\frac{\cos 2\epsilon}{\sin\epsilon}\,\sin(\delta_p - \dot{\Omega}t) - i'^2\cos\epsilon\Big] \tag{27}$$

$$\tilde{\dot{\epsilon}} = \Lambda\Big[\frac{1}{2}\sin\epsilon\,\sin(2\delta_p - 2\dot{\Omega}t) - i'\cos\epsilon\,\cos(\delta_p - \dot{\Omega}t)\Big] \tag{28}$$

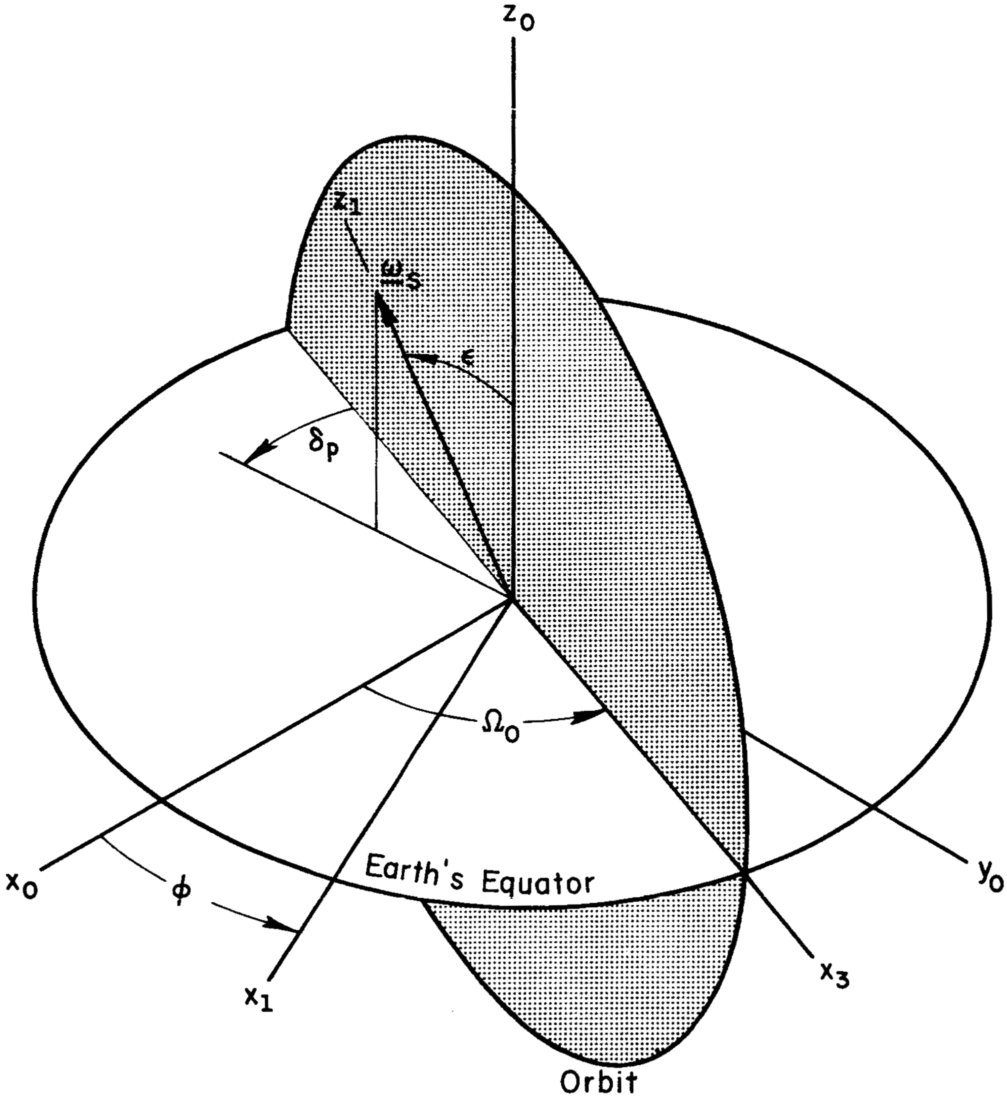


Fig. 3 Initial conditions for polar orbit.

These equations may be integrated with respect to time to give $\Delta\varphi$ and $\Delta\epsilon$ as functions of time:

$$\Delta\varphi = \frac{\Lambda}{2}\, t \left\{\cos\epsilon\left[1 - \sin 2\delta_p \left(\frac{1-\cos 2\dot{\Omega}t}{2\dot{\Omega}t}\right) - \cos 2\delta_p \frac{\sin 2\dot{\Omega}t}{2\dot{\Omega}t}\right] + 2i' \frac{\cos 2\epsilon}{\sin\epsilon}\left[\cos\delta_p \left(\frac{1-\cos\dot{\Omega}t}{\dot{\Omega}t}\right) - \sin\delta_p \frac{\sin\dot{\Omega}t}{\dot{\Omega}t}\right] - i'^2 \cos\epsilon\right\} \tag{29}$$

$$\Delta\epsilon = \frac{\Lambda}{2}\, t \left\{\sin\epsilon\left[\sin 2\delta_p \frac{\sin 2\dot{\Omega}t}{2\dot{\Omega}t} - \cos 2\delta_p \left(\frac{1-\cos 2\dot{\Omega}t}{2\dot{\Omega}t}\right)\right] - 2i' \cos\epsilon\left[\sin\delta_p \left(\frac{1-\cos\dot{\Omega}t}{\dot{\Omega}t}\right) + \cos\delta_p \frac{\sin\dot{\Omega}t}{\dot{\Omega}t}\right]\right\}. \tag{30}$$

If ϵ is allowed to vanish, these equations may give misleading results. In particular, Eq. (29) implies that $\Delta\varphi$ increases without limit as $\epsilon \to 0$. However, it must be remembered that φ and, therefore, $\Delta\varphi$, are undefined if $\epsilon = 0$ because the gyro spin axis becomes coincident with the z_o- axis, as can be seen in Fig. 3.

It will be seen later, that for practical purposes, the nodal regression rate, $\dot{\Omega}$, should be less than 45 degrees per year and, therefore, Eq. (26) indicates that for 400-700 mi orbits, i' must be no larger than 1^o or .017 radian. Consequently, for such slow regression rates, Eqs. (29) and (30) may be simplified by dropping the terms containing i'. The simplified expressions are

$$\Delta\varphi = \frac{\Lambda}{2} \cos\epsilon \left[1 - \sin 2\delta_p \left(\frac{1-\cos 2\dot{\Omega}t}{2\dot{\Omega}t} \right) - \cos 2\delta_p \frac{\sin 2\dot{\Omega}t}{2\dot{\Omega}t} \right] t \tag{31}$$

$$\Delta\epsilon = \frac{\Lambda}{2} \sin\epsilon \left[\sin 2\delta_p \frac{\sin 2\dot{\Omega}t}{2\dot{\Omega}t} - \cos 2\delta_p \left(\frac{1-\cos 2\dot{\Omega}t}{2\dot{\Omega}t} \right) \right] t \,. \tag{32}$$

Some typical curves are plotted in Figs. 4 and 5 to show the variations of $\Delta\varphi$ and $\Delta\epsilon$ as functions of the initial misalignment angle, δ_p, and the nodal regression angle, $\dot{\Omega}t$. In these curves, the nondimensional parameters $\Delta\varphi/\Lambda t \cos\epsilon$ and $\Delta\epsilon/\Lambda t \sin\epsilon$ have been plotted. These curves illustrate the need to keep the nodal regression rate and δ_p as small as possible to avoid large values of gravity gradient precession.

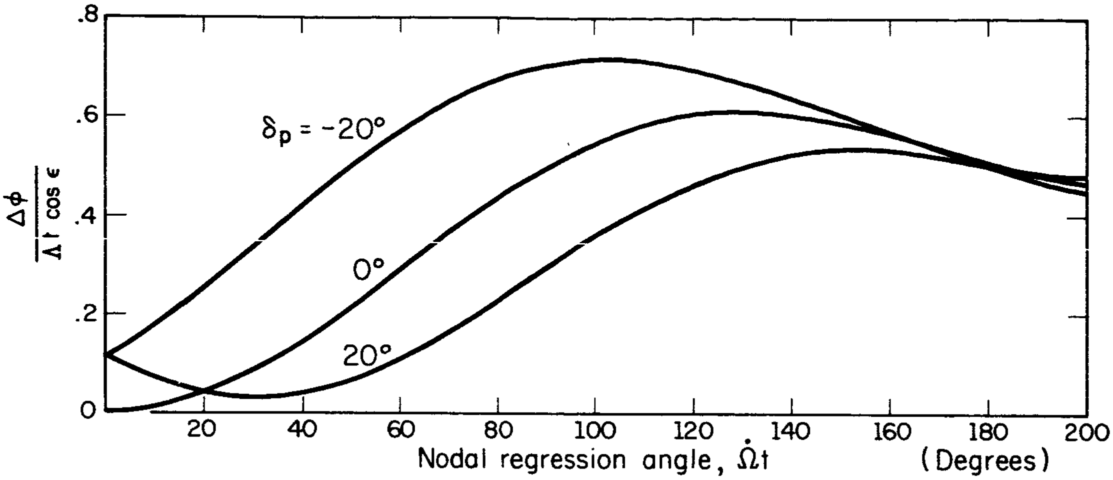


Fig. 4 Gravity gradient precession for regressing polar orbit; normalized equatorial plane component vs. regression angle.

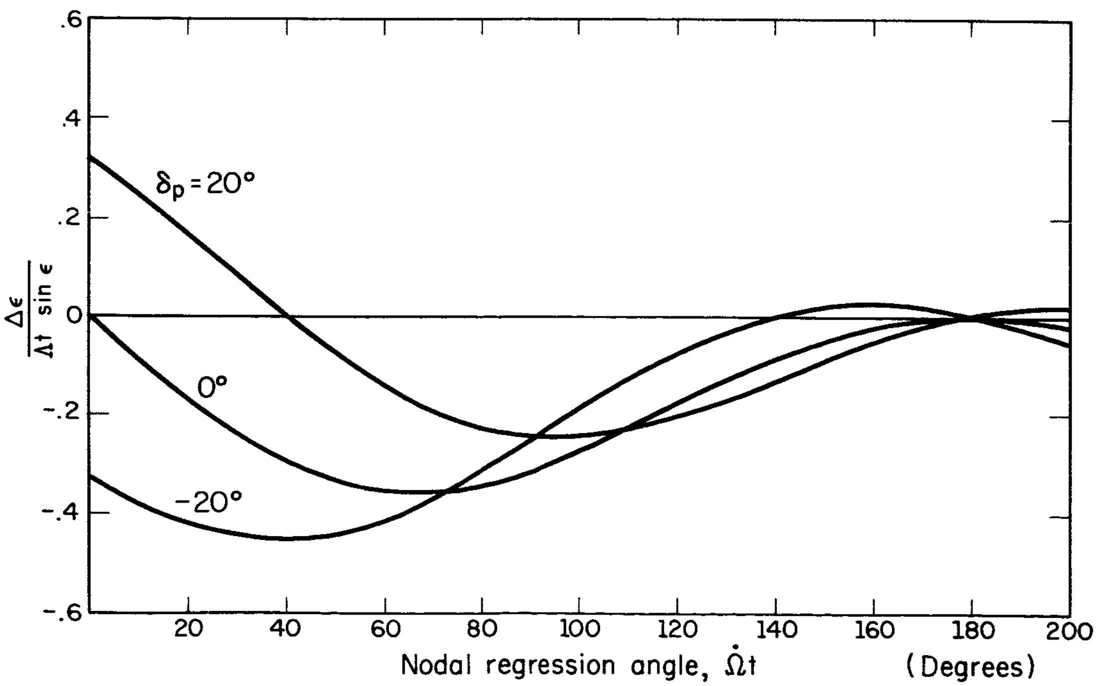


Fig. 5 Gravity gradient precession for regressing polar orbit; normalized component in plane of earth's pole and gyro spin axis vs. regression angle.

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